GIOVANNI E. CORAZZA - SERGIO AGNOLI - SARA MARTELLO
Bologna

COUNTERPOINT AS A PRINCIPLE OF CREATIVITY: EXTRACTING DIVERGENT MODIFIERS FROM *THE ART OF FUGUE* BY JOHANN SEBASTIAN BACH

In this article, we would like to share with the reader our belief that musical composition can be seen as a paradigmatic representation of the creative thinking process. This has direct consequences in terms of both theoretical interpretations and practical tools and techniques. Indeed, composition amounts to the objectification of a personal creative path followed by an individual, containing motivational, emotional, cognitive, and cultural elements. The striking discovery is that similar creative paths and principles can be found in very different domains, from art to science and technology. At first sight, creativity could appear as the reign of chaos, where different elements combine in an inspired but indefinite way, to generate a valuable product, an idea that was not foreseen *a priori*. In fact, creative thinking is a complex but explainable process, subject to the influence of numerous forces, internal and external to the individual, which determine the nature and the structure of the process itself. The classic compositional and executional techniques in music are clear examples of structure in the generative process, and are dictated by shared principles and methods, which of course depend on style, aesthetics, and historical period. This certainly does not reduce the process to the mere application of techniques, but it does imply that the generation of ideas is driven by codified methods, which help creators retrieve, combine, and move through conceptual elements. The contrapuntal technique, in particular, is at the same time a paradigmatic and a beautiful representation of the very nature of the creative thinking process. Apparently, it could be thought of as a set of well-known and relatively straightforward methods for the composition of polyphonic music; in reality, counterpoint can be read as a representation of the intrinsic impetus of the creative process, i.e., the combination and juxtaposition of opposite elements to go beyond established knowledge and enable the generation of new and valuable ideas. Johann Sebastian Bach is undoubtedly regarded as one of the most prominent counterpoint composers; what may be less obvious is the realization that his influence exceeds the already wide domain of music. Indeed, in the following we intend to show that today his masterpieces can be exploited to extract specific techniques for the generation of ideas in disciplined creative thinking processes, in different domains. This potential derives from the fact that the harmonic combination of contrasting

forces is indeed a structural dimension of the generative process, and hence it is inherently related to the nature of creativity. As a matter of fact, this idea has been the subject of philosophical exploration across the ages, as we will briefly describe in the following paragraphs.

Pythagoras and his school of thought were among the first to theorise that harmony derives from the equilibrium between contrasting elements[1] and not from the absence of contrasts. In Greek mythology, the beautiful goddess Harmonia was the daughter of Ares, god of war, and Aphrodite, goddess of beauty and love; she was the deity who connected extreme opposites. The very etymology of the word counterpoint – *punctus contra punctum* – represents the idea of contrasting entities, which, in Pythagorean terms, is at the origin of all elements: odd and even, light and darkness, rest and motion, and the list continues. In this sense, counterpoint could be seen as a translation of the very essence of nature in music. The coexistence of opposing forces in art is well represented in Nietzsche's notion of duality.[2] He explained that the flow of dramatic art, and its fascination, could be essentially described by the coexistence of two distinct "drives": the Apollonian form (order and measure) and the Dionysian form (chaos and illusion). Although these two forces are conflicting, they are far from being incompatible. The notion of duality implies that these two drives are actually inseparable forces, tied together since the origin of the world and belonging to each other. Nietzsche explains their coexistence through the concept of reconciliation of opposites, i.e., the acceptance of opposition as a constituent element of the nature of artistic creativity. Interestingly, Nietzsche himself acknowledges that musical counterpoint is a fitting analogy for the harmony of opposites. A more recent philosophical explanation of the generative power that can be unleashed by bringing together apparently incompatible realities can be found in the Act of Creation by Arthur Koestler (1964), where the concept of bisociative thinking is introduced. According to Koestler, bisociation is the fundamental mechanism for the generation of novelty, guided by sub-conscious processes and requiring superflexibility in order to accommodate the juxtaposition of incompatible elements, such as opposites.[3]

The relationship between contrasting concepts has also been the subject of linguistic and semiotic analysis. In 1966, Greimas developed the actantial model, an abstract tool to analyse real or narrative action: the actants are the

---

[1] ARISTOTLE, *Metaphysics, Book I-9*, trans. by H. Tredennick, Cambridge, Mass., - London, Harvard University Press - Heinemann, 1933, 983b-990a.

[2] F. NIETZSCHE, *The Birth of Tragedy or Hellenism and Pessimism*, trans. by W. M. A. Hausmann, in *The Complete Works of Friedrich Nietzsche, The first Complete and Authorised English Translation*, ed. by O. Levy, London, George Allen & Unwin LTD, 1923, I, pp. 13-17.

[3] A. KOESTLER, *The Act of Creation*, London, Hutchinson & Co. LTD, 1964, p. 659 sg.

mutually inter-related structural elements that develop the narrative sequence. Interestingly, actants are often interconnected through a relation of opposition: for example, a story develops thanks to a protagonist actant, e.g. a prince, who is defined by his opposition to another character, e.g. a dragon. Greimas emphasized that what matters are not the qualities of the isolated characters, but the functional relationships between them: it is this tension that generates the intended pathos. It should be noted that contrasting forces can also interact within a single character, as opposites that generate a creative development in the personal story of that character.[4] From a semiotic perspective, in 1968 Greimas and Rastier developed what can be described as the square of opposite entities: the *semiotic square*,[5] shown in Fig. 1. For this analytical tool, which can be applied in any domain, the concept of opposition is not considered as an absolute value, but as having several levels and degrees, with different types of contrast. The semiotic square is built by starting from a main focus element, identified as term A, mapped into the upper-left corner of the square. This element can be seen to stand in a relationship of contrariety with term B, reproduced in the upper-right corner of the square. At the same time, term A is in contradiction with term not-A, which occupies the lower-right corner. Finally, term B is in contradiction with term not-B, appearing in the lower-left corner. In this way, we obtain a number of possible relationships between opposites: A versus B, not-A versus not-B, A versus not-A, B versus not-B. All of these contrasts can be used not only to analyse a work of art, but also to generate ideas: for example, in Fig. 1 we chose to map canonical (Apollonian) approaches to composition as term A, from which we derived chaotic (Dionysian) approaches as term B. Therefore, the contrariety A vs. B corresponds directly to the Nietzsche's concept of duality or to a line in Pythagoras' table of opposites. At the same time, the contrariety between not-A and not-B leads to similar, but actually much softer, contrasts in the form of inclusion of small exceptions into musical composition. Therefore, opposition is not an absolute relationship, but it can also acquire different shades of grey.

---

[4] T. HAWKES, *Structuralism and Semiotics*, London - New York, Routledge, 2003, pp. 69-76.

[5] L. HEBERT, *Tools for Text and Image Analysis. An Introduction to Applied Semiotics*, University of Quebec at Rimouski, 2001, p. 48 (e-book available at *http://www.revue-texto.net/Parutions/Livres-E/Hebert_AS/Hebert_Tools.html*; 28th Dec. 2014).

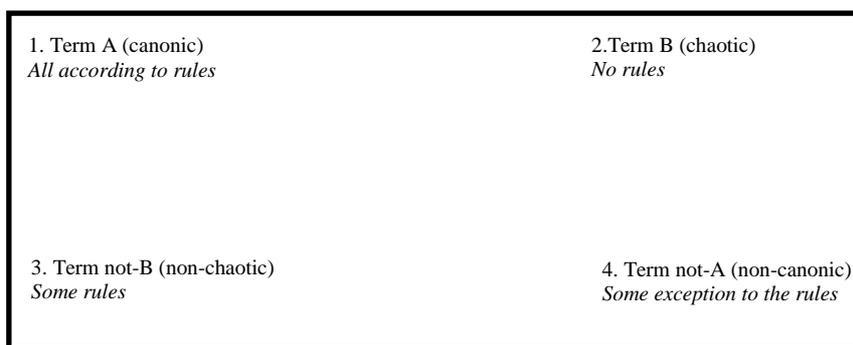|	
1. Term A (canonic) *All according to rules*	2. Term B (chaotic) *No rules*
3. Term not-B (non-chaotic) *Some rules*	4. Term not-A (non-canonic) *Some exception to the rules*

Fig. 1 – *The Semiotic Square, with application to musical composition.*

Apollonian and Dionysian drives were very relevant notions also in 19th century biology,[6] where scientists spoke of fundamental and strong impulses that permeate the basic biological functions in all living organisms. In particular, in regard to human behaviour, these forces were interpreted as contrasting elements inside our minds, i.e. those irrational and rational elements that followed different, but converging, evolutionary paths. In the following century, psychology came back to these contrasting elements in order to explain certain attitudes and actions of individuals and groups.[7] Indeed, both Apollonian and Dionysian elements play a key role in explaining both overt and covert behaviour. The Apollonian form stands for the cognitive, objectifying elements, while the Dionysian form represents the emotional, figurative elements. In spite of the cognitive revolution, which, since the post-World War II period, tried to set the focus of the psychological science exclusively on clear, objectifiable cognitive phenomena, modern psychology is deeply influenced by an integrated view of rational and emotional elements. Affective factors are not only the core of the human psyche, but also allow us to understand the integrated nature of cognition itself. Today, the coexistence of opposite forces has definitely found a central role in the study of the creative thinking process. Even in neuroscience, explanations of creativity[8] have found that the interaction of cognitive skills (mainly located in the frontal and temporal lobes) and emotional elements (residing in the limbic system) contains the nucleus of idea generation. In this essay, we have chosen to focus our attention on the functional level, which in psychological terms

---

[6] W. COLEMAN, *Biology in the Nineteenth Century. Problems of Form, Function, and Transformation*, Cambridge, Cambridge University Press, 1977.

[7] J. B. WATSON, *Psychology as the Behaviorist Views It*, «Psychological Review», XX, 1913, pp. 158-177; S. FREUD, *The Psychopathology of Everyday Life*, Authorized English Edition, with introduction by A. A. Brill, London, T. Fisher Unwin LTD, 1922.

[8] A. W. FLAHERTY, *Frontotemporal and Dopaminergic Control of Idea Generation and Creative Drive*, «Journal of Comparative Neurology», CDXCIII, 2005, pp. 147-153.

corresponds to the description and understanding of the high-level mechanisms that underlie the processes instantiated within the human mind. It should be noted that even contemporary philosophers are working on this same functional level, which shows that times are ripe for approaches that go well beyond mere speculation about creativity as a mysterious process. In *Creatività*, Garroni describes creativity in terms of a self-organizing model within a complex system which he calls "nature": referring to the creative process, he uses the term "legality" to define the application of general and recognisable laws, in this case the set of regularities that let humans perceive, interact and create in what he calls "nature".[9]

The DIMAI model for the creative thinking process

The model we developed[10] is a functional model for the creative thinking process, identified as DIMAI, which stands for Drive, Information, Movement, Assessment, and Implementation, and representing the constituent mental states in the generation of ideas. As we will briefly show in the following paragraphs, the DIMAI model is based on the presence and interaction of contrasting forces: we could go as far as saying that DIMAI represents counterpoint within the human mind. Modelling the creative thinking process has been a very productive area of scientific research for about a hundred years,[11] and the DIMAI model is a recent addition that makes claims of generality, for its ability to be transformed into other models under specific assumptions. The DIMAI model is graphically represented in Fig. 2.

This model is intended as general and domain-agnostic. In particular, it has been applied in technical domains, for example to provide a new interpretation of the set of tools for Inventive Thinking in technology that goes under the name of TRIZ.[12] Here we will try to describe DIMAI with special regard to music, but we urge readers to maintain full awareness of the fact that common principles underpin creativity in all domains. The DIMAI model involves horizontal and vertical dimensions: the five mental states are

---

[9] E. GARRONI, *Creatività*, Macerata, Quodlibet, 2010.

[10] G. E. CORAZZA - S. AGNOLI, *DIMAI: A Universal Model for Creative Thinking*, in «Marconi Institute for Creativity Internal Report», I, 2013.

[11] *Encyclopedia of Creativity*, ed. by M. A. Runco and S. Pritzker, 2nd ed., Amsterdam, Elsevier, 2011; T. LUBART, *Models of the Creative Process: Past, Present and Future*, «Creativity Research Journal», XIII, 2001, pp. 295-308; G. WALLAS, *The Art of Thought*, New York, Harcourt Brace and Company, 1926; M. D. MUMFORD - K. E. MEDEIROS - P. J. PARTLOW, *Creative Thinking: Processes, Strategies, and Knowledge*, «The Journal of Creative Behavior», XLVI, 2012, pp. 30-47.

[12] S. AGNOLI - G. E. CORAZZA, *TRIZ as seen through the DIMAI Creative Thinking Model*, in *TRIZ Future Conference 2013*, Proceedings of the 13th ETRIA world TRIZ future conference 2013 (Paris, 29th-31st of October 2013), ed. by A. Aoussat *et alii*, Paris, Arts et métiers ParisTech, 2013, pp. 23-33.

interfaced by selection gates and can be vertically split into two modalities. The convergent modality is a tendency of the mind to move towards a single pattern showing "the best possible match", and hence it is selective in nature; the divergent modality, on the other hand, is the tendency to move towards any and all patterns that exhibit a "sensible match", i.e. towards all possible alternatives, and is therefore exploratory in nature. These convergent and divergent modalities are the contrasting and opposing forces that act within our mental states to produce, generate, and innovate.
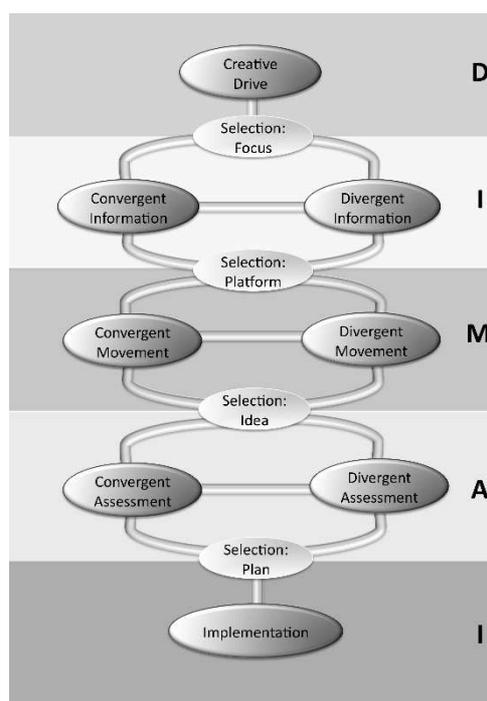

Fig. 2 – *The DIMAI model.*

Let us now consider the five mental states in detail. The *Drive* state entails the selection of a focus area, where new ideas shall be generated, along with the motivation to dedicate effort to the always problematic and risky activity of challenging the state of the art. In the case of composition, this would correspond to the selection of a specific genre and style, in which new music should be generated, along with the spark for approaching the exercise with energy and will. The stage is completed when a single, refined *Focus* area is selected among a series of possible areas of attention. The next state, *Information*, involves the collection or recollection of all the knowledge elements that can be used to generate new ideas within the focus area. We can take these knowledge elements as items related to the focus area in a semiotic perspective: referring to the trichotomies theorised by Peirce, relevant information can be

identified using relationships of similarity (iconic reference), contextual contiguity (indexical reference), or law (symbolic rule).[13] Obviously, expertise plays a major role in collecting relevant information. Years upon years of application and study pay off in terms of abundance of elements that are activated once the task is started. In general, we describe this kind of information as *convergent*, i.e. relevant to the problem at hand. However, as can be inferred from the diagram, it is also possible to introduce into the process a few elements of information that are not strictly relevant, and are therefore classified as *divergent*. Clearly, there is no *a priori* rationale for introducing such disturbing elements, other than the awareness that they are the key for deviating from the established paths, and hence the door to originality. Divergent elements are inspiring: they can come from nature, from accident, from randomness. But they can also be introduced systematically, by transforming convergent information with the help of tools that are referred to as *divergent modifiers*. These tools are so important that the entire message of this article could be condensed into the following sentence: *contrapuntal techniques act as divergent modifiers to be applied onto previously established musical elements.* Harmonious originality is therefore the artistic product of the coexistence between relevant/irrelevant, standard/non-standard, convergent/divergent musical elements: the notion of duality revisited. Are some modifiers more powerful than others? This is a fundamental question, which cannot be easily answered: let it suffice to say that the number of possibilities is virtually infinite, and the reason is that "incorrect" divergent information cannot really exist, since by definition it includes anything which is not strictly relevant to our focus. The disciplined composer must proceed by selecting a *Platform*, consisting in a set of convergent elements and possibly a single divergent modifier, which enables him/her to enter the mysterious world of the next mental state, *Movement*: here, judgement must necessarily be suspended, in order to let the mind explore and travel through the intricate network of alternative musical paths, moving from one idea to the next through insight, analogy, and metaphor. It is here that the coexistence of opposites exerts its power by leading the mind into unexplored places, which would have never been visited in a condition of safety within the space of commonly accepted knowledge and facts. Movement itself can either be convergent or divergent, depending on the thinking style adopted: systematic search for the best possible consequence, or free exploration of suitable alternatives. Clearly, movement is not always successful when it comes to generating a new musical idea: the end result may be nothing but an elicitation of pre-existing patterns.

---

[13] CH. S. PEIRCE, *Collected Papers of Charles Sanders Peirce*, vols. 1-6, ed. by Ch. Hartshorne and P. Weiss, Cambridge, The Belknap Press of Harvard University Press, 1931-35; vols. 7-8, ed. by A. W. Burks, Cambridge, Harvard University Press, 1958.

Even in this case, the exploration is still a valuable exercise, as it might help us look at the existing reality with fresh eyes, having discovered a different connection with our established knowledge. While we demand that judgement be suspended in order to reach deeper in our mind, the *Assessment* of the *Idea* we are generating is an almost irresistible temptation that constantly crops up throughout the thinking process. *Assessment* is our next state in the DIMAI model, and, rather surprisingly, it can again take the convergent and divergent form: in the former case, we judge the novel musical idea with respect to our initial focus, while in the latter we feel free to check whether we have produced a kind of music that is aesthetically viable to our ears, and in this way we can find what we were not looking for, as a consequence of serendipity. Clearly, although our diagram represents the states in a sequential and orderly fashion, coexistence and iteration are at a prime position in our minds. This applies specifically to *Movement* and *Assessment*, to which we can go back repeatedly and rapidly as many times as necessary, not only in composition but also in improvisation. This process is particularly well described by the so-called Geneplore model.[14] Finally, once the composer is satisfied with an idea, he/she can formulate a *Plan* and enter the final state, that of *Implementation*, turning the musical idea into reality and submitting it to the judgement of others. In this respect, we know from the experience of even the greatest composers of all times that the higher the originality, the stronger the reaction will be on the part of critics. This is why creativity requires courage and endurance, without which talent remains trapped into the prison of untapped potential. Let us now come back to counterpoint as a classic compositional technique, now that we have set the premises to re-interpret it as an disciplined tool for the introduction of divergent information into the compositional practice, through the modification of convergent musical elements.

Counterpoint techniques as divergent modifiers in fugue compositions by Bach

Starting from consonances and dissonances as the opposite but interdependent elements that constitute harmony, the Austrian composer and music pedagogue Johann Joseph Fux wrote his famous dialogue on the theory of contrapuntal music between Aloysius, the venerable master, and Josephus, the student. According to the words of Aloysius, the final objective of harmony is to give pleasure to the listener, and this feeling arises from what the author refers to as a "variety of sounds".[15] As noted before, the term *counterpoint* derives from *punctus contra punctum*, meaning that, as Aloysius explains, it represents the compositional principle whereby *a note is placed "against" another.*

---

[14] R. A. FINKE - TH. B. WARD - S. M. SMITH, *Creative Cognition: Theory, Research, and Applications*, Cambridge, Mass. - London, MIT Press, 1992.

[15] J. J. FUX, *Gradus ad Parnassum*, in *The Study of Counterpoint from Johann Joseph Fux's "Gradus ad Parnassum"*, trans. and ed. by A. Mann, New York, Norton, 1971, p. 21.

Counterpoint therefore requires a search for balance between the different and opposite musical forces that permeate each voice, and every note in each voice, with the aim of creating what we can describe as harmonic equilibrium. Counterpoint can be used to produce several musical structures, but this article only concentrates on the fugue, which is more than adequate for our purposes. In particular, we chose to focus our attention on the compositions that make up the masterpiece entitled *The Art of Fugue* by Johann Sebastian Bach, conceived in the years from 1742 to 1747. The initial work consisted of twelve fugues (identified as "Contrapunctus") and three canons, while the final, revised version, which includes fourteen fugues and four canons, was published about a year after Bach's death in 1751.[16]

Our purpose is to illustrate how the great Baroque genius, in his search for an exemplary and exhaustive exploration of possible compositional techniques for the fugue (a beautiful exercise of divergent thinking), started from a simple, basic subject (convergent information) and applied contrapuntal techniques (divergent modifiers) to move from one idea to the next and find valuable solutions to the challenges that he set himself. Citing Williams: «In the complex play of themes or canons, one has the impression that Bach is answering the initiated listener's question, "How is he going to solve that problem?" At the same time, the harmony of the theme is so fundamental and reliable that there is no limit to the number of good countersubjects it can prompt».[17]

To be more precise, Bach's refined creative focus in DIMAI terminology was the composition of fugues as variations on the following main subject in D minor:

Fig. 3 – *The Theme of the Art of Fugue.*

The compositions presented in *The Art of Fugue* are organized by increasing level of complexity, and the divergent modifiers are added up as the work develops. Clearly, in each Contrapunctus many compositional elements could be extracted and added to our analysis, but we shall limit ourselves to considering a single distinctive element for each of the fugues. Starting with Contrapunctus I, it can be noted that the subject is presented in the contralto voice, while the soprano answers the opening statement with an adjusted transposition in the A minor key, which is fictional but still effective. The divergent modifier principle that can be extracted here is that of *translation*:

---

[16] P. WILLIAMS, *J. S. Bach. A life in Music*, Cambridge, Cambridge University Press, 2007, p. 248.

[17] *Ibid.*, p. 251.

starting from an item of convergent information, we introduce a divergent element by semantic transformation, according to a new reference system of symbols, i.e., in this particular case, a new tonal reference. *Translation* is therefore our first divergent modifier inspired by the work of Bach. In order to show that these Bach-inspired divergent modifiers enjoy wide applicability, we will provide short and simple examples of use in two unrelated domains: education and computer science. As an example of the use of *translation* in the field of education, a teacher could ask students to merge the study of music with that of a foreign language, by translating the work of original critics of Bach. As a second example in the field of computer science, an idea generated with the help of this divergent modifier could be to create a tool for the automatic translation of software code to enable compatibility across all possible computing platforms.

Moving on to Contrapunctus II, Bach enters the time domain with rhythmic variations on the subject to create the impression of a swinging motion, and a definite sense of freshness. Now, this can be applied to any convergent concept in the form of *compression* and *expansion* of matter. This technique could inspire a learning approach in literature: for a specific chapter of a book, the teacher could ask students to produce "rhythmic variations" on the content, for example by summarizing odd paragraphs and expanding even paragraphs. In computer science, the human-computer interface could be automatically customized to suit individual users, for example by sensing their distance from the display and expanding the font size accordingly.

The use of *inversion*, which is a classic contrapuntal technique, is evident in Contrapunctus III. Indeed, inversion is also a classic divergent modifier, which substitutes an element of accepted knowledge with its opposite, typically leading to a paradoxical situation. This paradox, once reconciled with the world as we know it, can leave as a by-product a novel idea. If we take the assertion "The teacher instructs the pupil" and invert it, we obtain "The pupil instructs the teacher", which of course does not perfectly mirror reality. However, this can lead to the idea of assigning homework on new (yet unexplained) topics, about which the students are expected to give short classes, with possible examinations of the teacher himself/herself. Following a similar path, "The computer runs the program" leads to "The program runs the computer", an apparently strange statement which can be used as a platform to move to the idea of remotely controlling machines through smart communication infrastructures.

The part devoted to the individual fugues ends with Contrapunctus IV. If we focus on the voices entry pattern in Contrapunctus I, the fugue follows a circular dialogue between voices: C (contralto), S (soprano), B (bass), and T (tenor), while in Contrapunctus IV the dialogue starts from S and continues with C, T, and B. This kind of arrangement introduces what we can describe as the *shuffle* modifier, whereby a sequence of elements is reshuffled into a new order. If we want to apply this idea to educational approaches, we could

imagine a teacher who presents different sections of a short novel in a random order, so that pupils have to reconstruct the correct story: this device would be helpful in remembering plots, or learning about playing with time in narration. In the computer science domain, the *shuffle* technique is already applied in mp3 devices, where playlists can follow a random order, but we could take this idea further and imagine a game in which listeners have to guess the "best" next piece of music based on the previous ones.

The *overlap* divergent modifier is applied when two elements of convergent information are superimposed to form a new "distorted" element. This definition recalls the *stretto* contrapuntal technique, which is introduced in the exposition of Contrapunctus V: the various voices partially overlap and harmonize with the flowing timbre. Overlap could be a playful method for pupils to create a new story by using characters of one novel and the plot of another, e.g. to write a story on how Renzo and Lucia would have lived inside Moby Dick. The overlap modifier in computer science can be exploited to create multiple overlays of information about a situation, thus leading to layered virtual or augmented reality.

In Contrapunctus VI and VII we can notice the application of *diminution* and *augmentation* techniques, i.e. note durations that are proportionally diminished, or respectively increased, typically by factors of 2. These are in themselves divergent modifiers that can be easily applied to any domain. In education, using the *diminution* modifier could for example lead to the idea of halving the duration of lessons on the last day of the week, in order to let all subjects have a small space for synopsis, question/answers sessions, or homework explanation and assignment. In computer science, the *augmentation* modifier could be applied to the dynamic adaptation of computing resources depending on the remaining energy available in portable devices.

Contrapunctus VIII is a triple fugue, built on the interaction between three voices. Here the modifier can be defined as *multiple overlap*, an extension of the overlap modifier whereby three or more elements are superimposed. Clearly, the number of possibilities grows combinatorially with the number of elements that we overlap, leading to both richness and complexity.

The subject presentations in Contrapunctus IX alternate between D minor and its related keys using *invertible counterpoint*, in which parts are written to be interchangeable, so that each melodic line can function as a bass line as well. The invertible counterpoint technique can be associated to the classic *role-play* modifier, where an element "acts" or "behaves" according to the intrinsic features of another one. Using this modifier in education, subjects that are typically considered to be complementary can, for a limited period of time, play the role of the main subject, which could lead to deeper understanding and cross-fertilization. In the computer science field, we could imagine software and hardware elements exchanging features: hardware becomes soft (e.g.

wearable devices) and software becomes hard (e.g. pre-installed and hidden into everyday objects).

Contrapunctus X is a double fugue built on the *contrary motion* technique, which is another form of inversion, with opposite melodic directions across different voices. This inspires the *reverse* divergent modifier. Elements can be reversed in the up – down sense, but also backward – forward, considering space, time, action, motion, and also functionality as possible contexts of application. For example, the *reverse* modifier could be useful in teaching math: instead of showing how to solve a problem, the teacher could start from a solution and demonstrate how a problem is built, with growing levels of complexity. This is actually an extremely effective teaching technique. Reversing concepts has been used repeatedly in computer science: one of the recent, and perhaps more outstanding, examples is that of the TCP/IP protocol, the basis for the success of the Internet. This protocol reverses the concept of strict quality control at the lowest transmission levels by introducing the best effort approach at the highest application level.

Retrograde motion is the device applied by Bach in Contrapunctus XI, whereby melody notes are played backwards – which can be simply interpreted as the reverse modifier as applied to the time domain.

In Contrapunctus XII and Contrapunctus XIII all of the voices from the "rectus" fugue are inverted into the second fugue, and this is the reason why they are called mirror fugues. This compositional device leads to the *mirror* modifier, where the thinker has to look at convergent information as if it were reflected by a mirror: left becomes right and multiple elements can be mirrored at the same time. In the pedagogical field, a simple classroom exercise would be for right-handed pupils to write with their left hand, and viceversa, to enhance mutual understanding. Considering smartphone applications with touchscreens, we could think of games involving mirrored moves to visually teach dancing or playing an instrument.

We cannot avoid mentioning the final, and famous, unfinished fugue of Contrapunctus XIV, borrowing the concepts explained by Hofstadter in his dialogue between Achilles and the Tortoise.[18] The two symbolical characters talk about *Contracrostipunctus*, a compound word that results from the combination of the terms Contrapunctus and Acrostic. Just like Lewis Carroll did in his poems, Bach used an acrostic device to compose the structure of Contrapunctus XIV: he started from the letters of his last name, BACH, putting them in relation to the German music notation, in which B stands for B-flat and H for B. Can we consider this as a divergent modifier of some kind? The answer is negative, since there is no transformation of convergent information, and yet Bach was able to introduce divergent elements with no a priori justification.

---

[18] D. R. HOFSTADTER, *Gödel, Escher, Bach. An Eternal Golden Braid*, Harmondsworth, Penguin, 1980, p. 83.

This technique can therefore be interpreted as the introduction of a *random element*: indeed, the appearance of unforeseen elements is associated with numerous major advances in the history of science.

In conclusion, we hope that our analysis was able to convince the reader about the fact that Bach's beautiful fugue constructions all followed a common principle, i.e. the introduction of divergent modifications starting from elements of established knowledge. This corresponds in an astonishing way to the description of the Information mental state in the DIMAI model, whereby convergent and divergent information elements are brought together to produce a contrast, an opposition, a contradiction that generates a platform with a high potential for moving from idea to idea, in a search for originality and value, as perceived by individuals and society at large.